UNIT 1. The Nature of AtmosphericWind

What this unit is about

To be able to predict the performance of a wind turbine it is essential that a designer or wind site developer has a knowledge of the behaviour and structure of the wind itself. It is therefore necessary to understand the main causes of atmospheric wind and the statistical nature of wind.

This unit describes how the wind speed changes over different time periods; from seconds through to seasonal and yearly variations. It describes how wind speed changes with height above ground level and how it is affected by surface structure or topography.

Estimation of the long term wind speed distribution at a given site allows the annual energy capture for a given turbine to be determined. This unit describes how this can be done experimentally and through mathematical modelling.

What you will learn

After you have worked through this unit you will be able to:

- Explain the causes of atmospheric wind.
- Calculate how the wind speed varies with height above ground level for a given wind site.
- Describe the variation of wind speed with time in both the long and short term.
- Describe the general annual wind speed distribution.
- Model real wind speed data using the Weibull distribution.
- Use the Weibull distribution to calculate important quantities such as annual energy capture and optimum design wind speed for a given turbine.

Unit 1 Contents

1.1 The Causes of Atmospheric Wind

1.1.1 Solar Heating of the Atmosphere

Winds are large-scale movements of air masses in the atmosphere. These movements of air are created on a global scale primarily by differential solar heating of the Earth's atmosphere. Therefore, wind power can be thought of as an indirect form of solar energy.

Air at the equatorial regions is heated more strongly than at other latitudes, causing it to become lighter and less dense. This warm air rises to high altitudes and then flows northward and southward towards the poles where the air near the surface is cooler. This movement ceases at about 30°N and 30°S, where the air begins to cool and sink and a return flow of this cooler air takes place in the lowest layers of the atmosphere.

The areas of the globe where air is descending are zones of high pressure and where air is ascending, low pressure zones are formed. This horizontal pressure gradient drives the flow of air from high to low pressure, which determines the speed and initial direction of the wind motion.

In describing the direction of the wind, we always refer to the direction of the original wind. That is, a north wind is blowing *from* the north and is going *toward* the south.

The greater the pressure gradient, the greater is the force on the air and the higher is the wind speed. Since the direction of the force is from higher to lower pressure, and perpendicular to the isobars (lines of equal pressure), the initial tendency of the wind is to blow parallel to the horizontal pressure gradient and perpendicular to the isobars. The resulting wind is called the *gradient wind*.

For straight or slightly curved isobars the resultant wind is also called the *geostrophic wind*. However, as soon as wind motion is established, a deflective force is produced due to the rotation of the earth which alters the direction of motion. This force is known as the Coriolis force. **See Level 2: Page 4**

Level 2

The Rotation of the Earth - The Coriolis Effect

The Coriolis force explains the effect of the earth's rotation under a moving parcel of air. From a fixed vantage point in space air would appear to travel in a straight line. However, from the vantage point on earth it appears to curve. Therefore the Coriolis force is not actually a force at all but merely due to viewing a moving parcel of air from a rotating frame of reference. The basic effect is shown in figure 1.

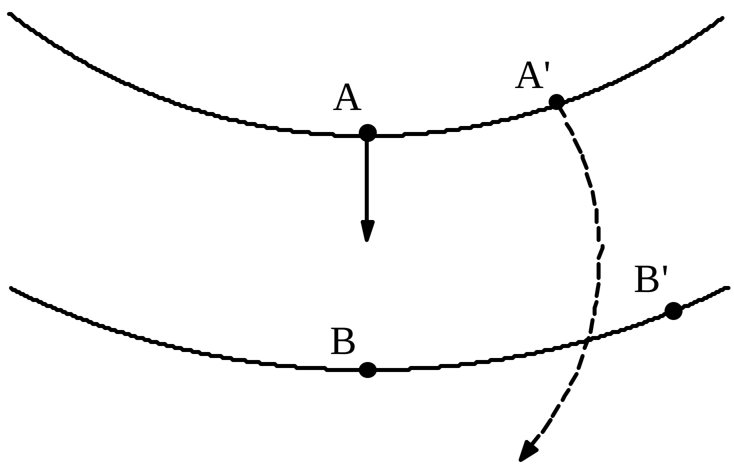

Figure 1: The Coriolis effect. The two curved lines are at constant latitude, with point B located directly south of point A. A parcel of air is moving south from A towards B. At some later time the two points on the surface of the earth will have moved to points A′ and B′ respectively looking from above, due to the earth's rotation. If we assume the speed of the air remains constant with respect to the ground, the direction will change because the of the earth's rotation under the parcel.

The circulation of the atmosphere is profoundly influenced by the rotation of the Earth (which gives a surface speed of approximately 1600km/hr at the equator, decreasing to zero at the poles).

It can be shown that the rotation causes the air flow from the equator to the poles to be deflected towards the east and the return flow from the poles to the equator to be deflected towards the west producing the so-called *Trade Winds*. North of latitude 30°N and south of latitude 30°S the atmospheric motion is characterised by westerly winds. When strongly curved isobars are present, a centripetal force must also be considered.

A simplified diagram showing the world's large-scale air movements is shown in figure 2.

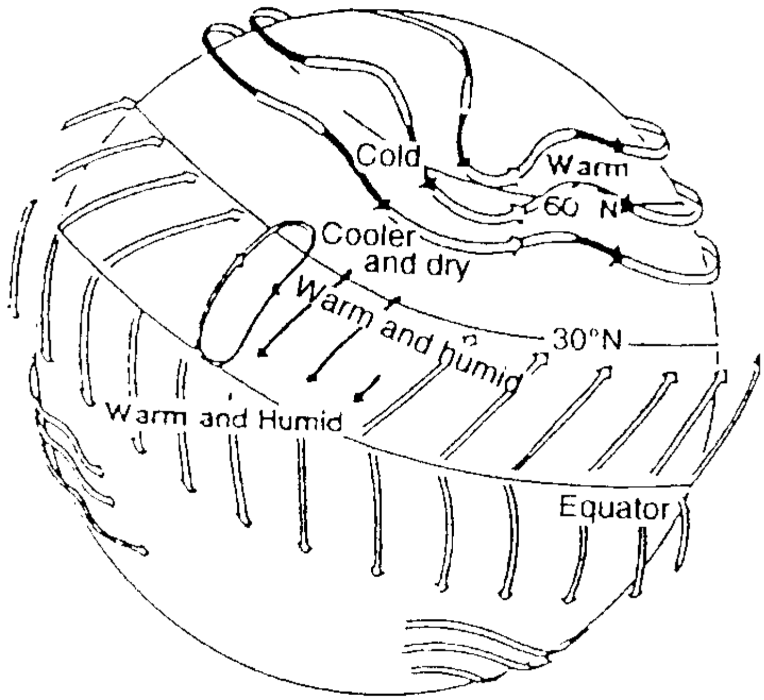

Figure 2: The General circulation of winds over the surface of the Earth

In addition to the main global wind systems there are also a variety of local effects. Differential heating of the sea and land also causes changes to the general flow. The nature of the terrain, ranging from mountains and valleys to more local obstacles such as buildings and trees, also has an important effect.

An explanation of weather systems and the effects they have on atmospheric pressure are given in the next section.

1.1.2 The Weather and Atmospheric Pressure

The atmospheric pressure at a point is the result of the column of air above that point. The air pressure can be measured using a barometer and is generally in units of bars, (although the SI unit for pressure is the Pascal measured in N/m^2, where 1bar = 10^5 Pascals). Atmospheric pressure is approximately 1 bar and barometers are usually calibrated in millibars (i.e. one thousandth of a bar). The average atmospheric pressure at sea level is about 1013.2mbar.

A typical weather map of Europe is given in figure 3 showing areas of high and low pressure and weather fronts is shown below. High pressure regions tend to indicate fine weather with little wind, whereas the low pressure regions indicate changeable, windy weather and precipitation. In the northern hemishpere, winds rotate anti-clockwise into low pressure regions (known as cyclones) and clockwise out of high pressure regions (known as anti-cyclones).

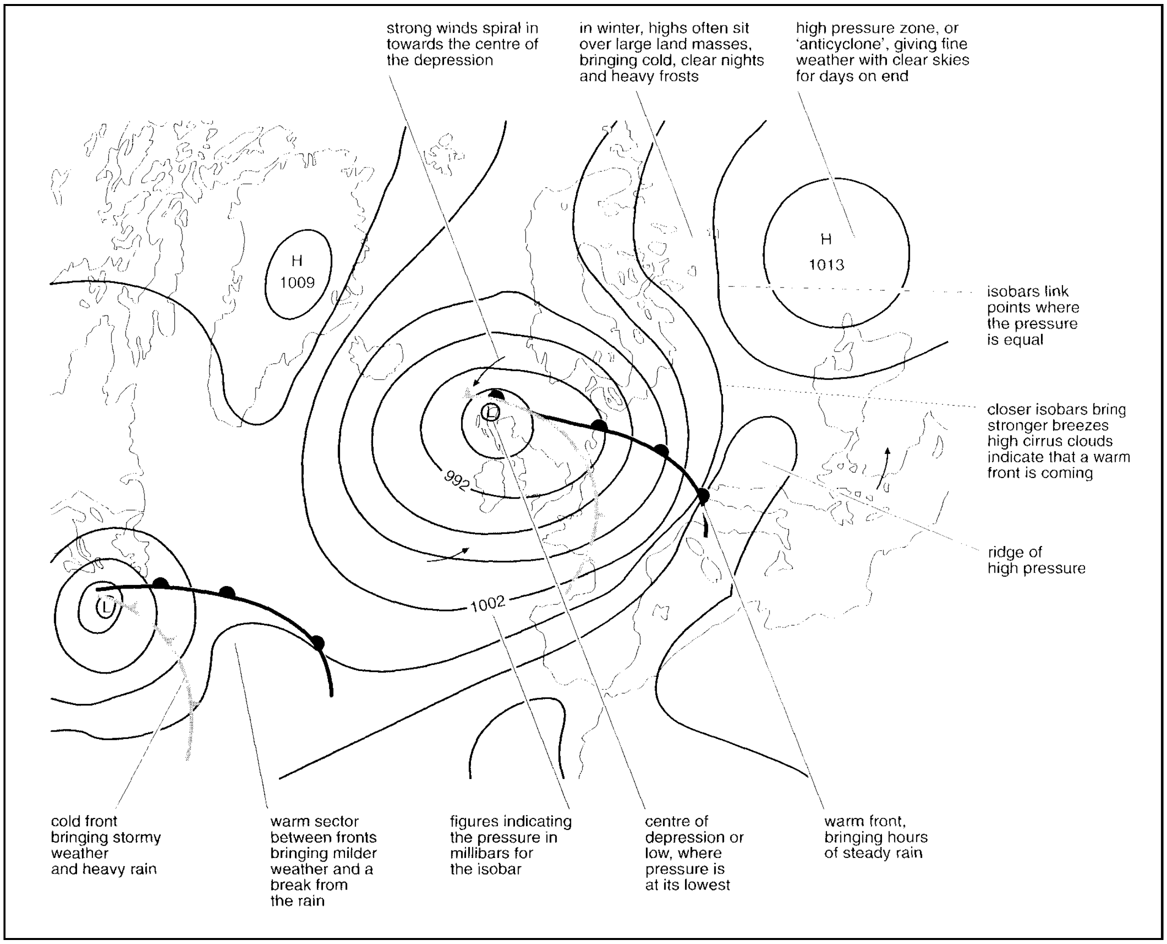

Figure 3: A typical weather map of Europe with an explanation of weather systems.

Article 1 gives a brief outline of the causes of wind and introduces some of the ideas that we will be looking at in detail in this unit and unit 2.

1.2 The Variable Nature of Wind

The wind speed at a given location is continuously varying. There are changes in the annual mean wind speed from year to year; changes with season (*seasonal*), with passing weather systems (*synoptic*), on a daily basis (diurnal) and from second to second (*turbulence*). Figure 4 shows typical wind speed variations over different time periods for a given location.

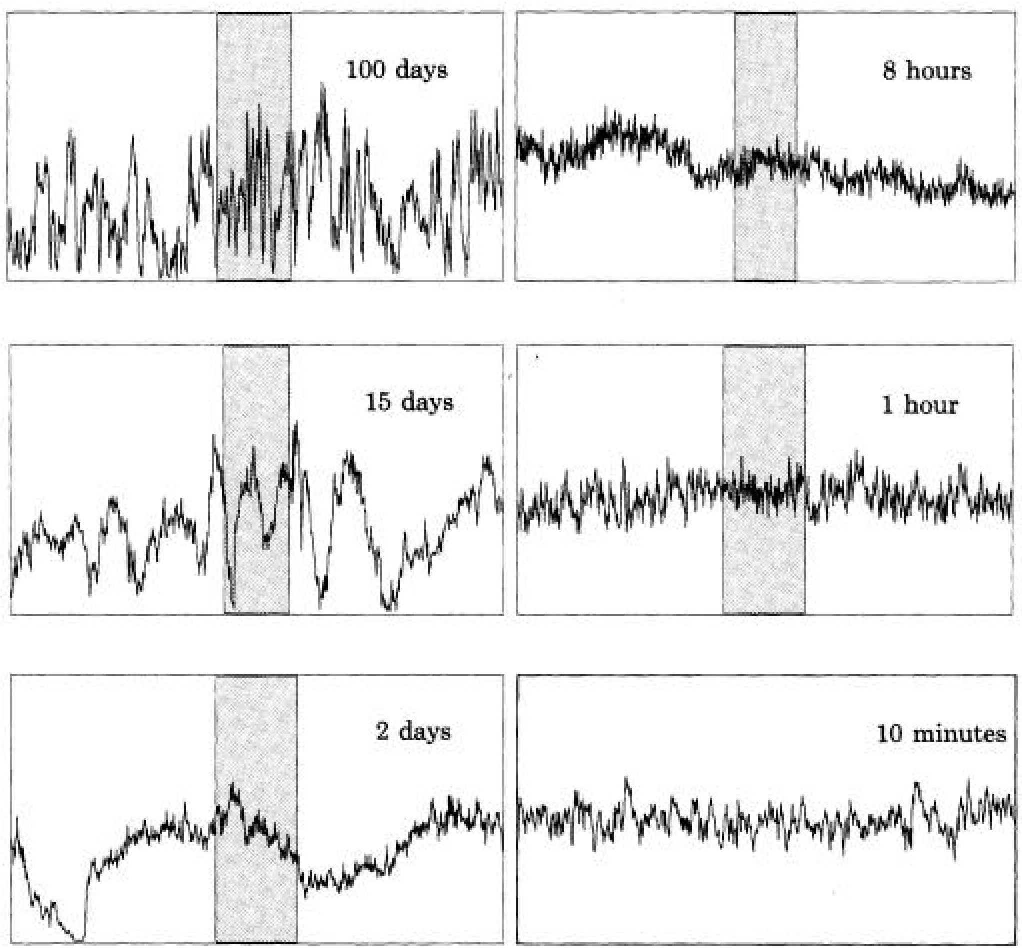

Figure 4: Variation in wind speed on different time-scales. The shaded area in the 100 day sample refers to the whole area in the 15 days sample, etc
By kind permission of Risø, Denmark

All these changes, on their different time scales, can cause problems in either predicting the overall energy capture from a site (annual and seasonal), for wind speed measurements (synoptic, diurnal and turbulence) and for wind turbine design (all time scales are important).

We can see that although the wind speed is continually changing it is most constant over a range of between 10minutes and 1 hour. If the averaging period for the mean wind speed is chosen to lie within this range, the synoptic and diurnal variations can be separated from those due to turbulence.

Generally, measurements are taken every 10 minutes to an hour and there are several good reasons for choosing an averaging period in this range.

1. It lies near the spectral gap and so stable averages will be obtained

2. It is short enough to reflect sharp sudden storms

3. If much shorter periods were used, data sets would be large and unmanageable

A typical plot of wind speed over a number of minutes is shown in figure 5. It can be seen that there is considerable variation in wind speed over the whole period and that significant variations occur over periods of a few seconds.

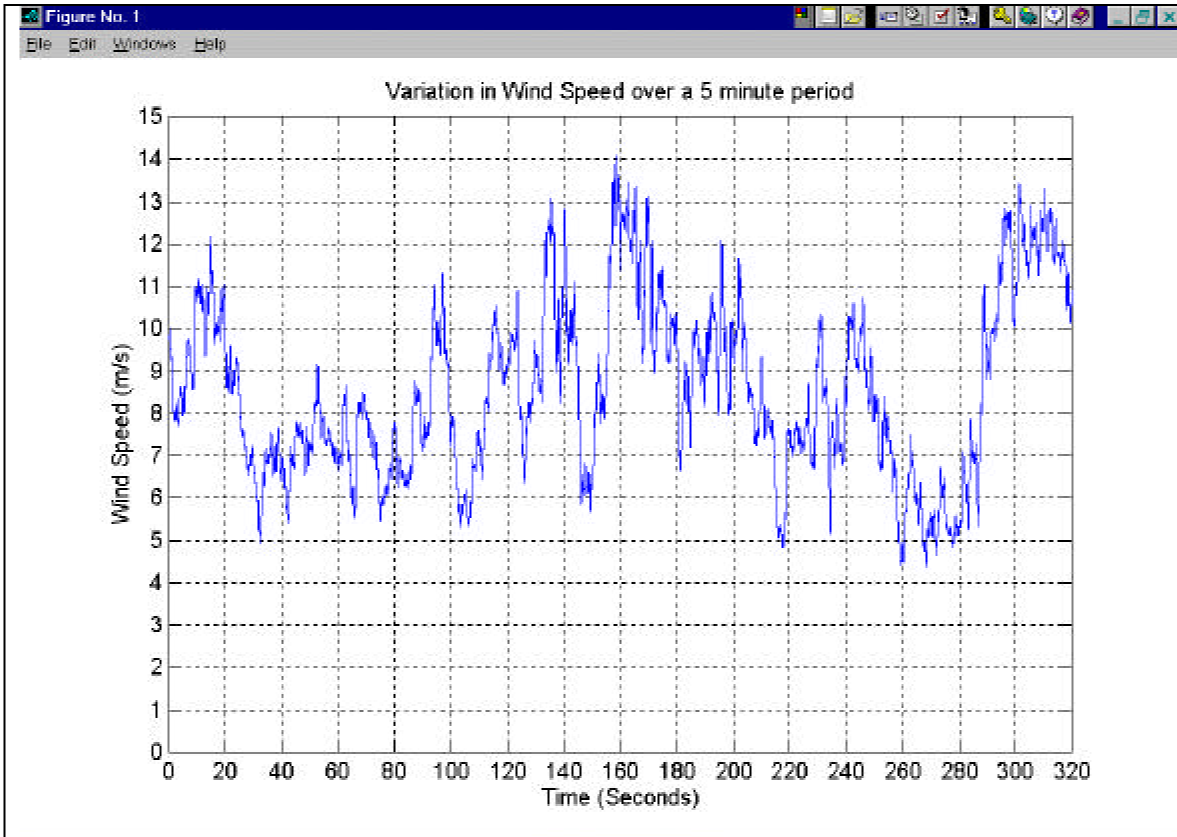

Figure 5: The wind speed over a 5 minute period (measured at the Rutherford Appleton Laboratory, UK May 1995). The figure has been generated from raw anemometer data using MATLAB.

The variations of wind speed over different time scales are well illustrated in a Van de Hoven Spectrum this is explained in **Level 2 page 9**.

Level 2

The Van de Hoven Spectrum

This effectively gives the amount of variation in the wind speed associated with a particular time-scale (or frequency).

The Van de Hoven spectrum is shown in figure 6. It can be seen that the dominant contribution comes from passing weather systems which occur typically over a few days which can result in extremely large changes in wind speed. These are the synoptic variations. The daily (diurnal) variation can also be significant (depending on the exact location), as is the turbulent variation with a time-scale of minutes to seconds.

The "Spectral Gap"

An interesting feature of the Van de Hoven spectrum is that there exists a 'spectral gap' - a period over which there is little variation - between say ten minutes and two hours.

The significance of this spectral gap is that, if the averaging period for the mean wind speed is chosen to lie within this range, the synoptic and diurnal variations can be separated from those due to turbulence.

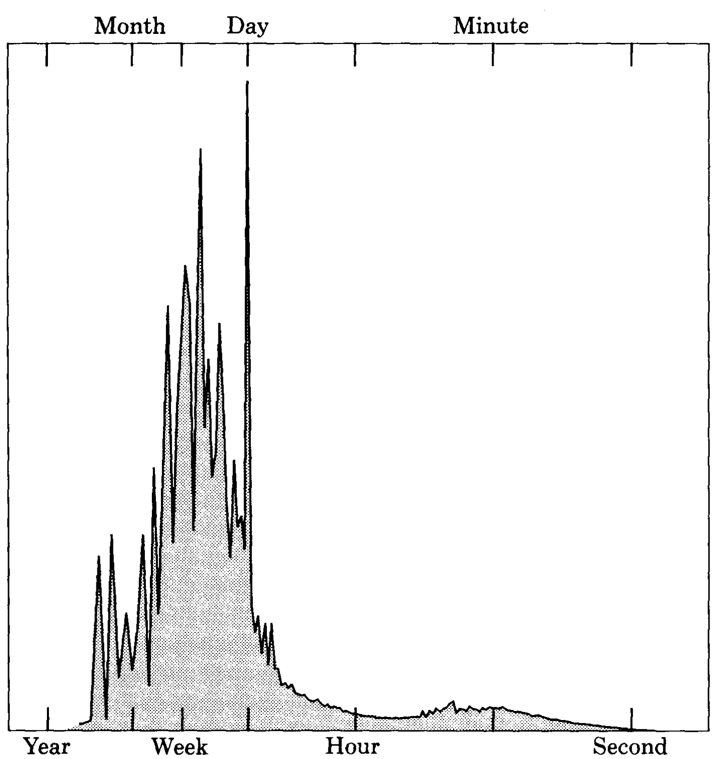

Figure 6. The Van de Hoven Spectrum showing the amount of variation in wind speed on a particular time-scale.
By kind permission of Risø, Denmark

1.3 The Boundary Layer

1.3.1 Variation of Wind Speed with Height

The strongest, steadiest and most persistent winds occur in bands some 10km above the earth's surface, known as the *jetstream*. Wind turbines, however, are presently limited to the lowest 100m of the atmosphere. At these heights the wind is strongly affected by the surface, through friction, and hence wind speeds are lower. The region below about 1-2km, where the wind is affected by friction is known as the planetary boundary layer.

The boundary layer refers to the lower region of the atmosphere where the wind speed is retarded by frictional forces on the Earth's surface. In general, the wind speed is nominally zero at ground level, in accordance with the 'no-slip' condition, and increases steadily with height.

The lower layers of air tend to retard those above them until the shear forces (i.e. the forces parallel to the ground) are gradually reduced to zero. The change of wind speed with height is known as the *wind shear*. A diagram showing the Earth's boundary layer, the various names for the different regions within the lower atmosphere and an indication of how the mean wind speed typically varies with height is shown in figure 7.

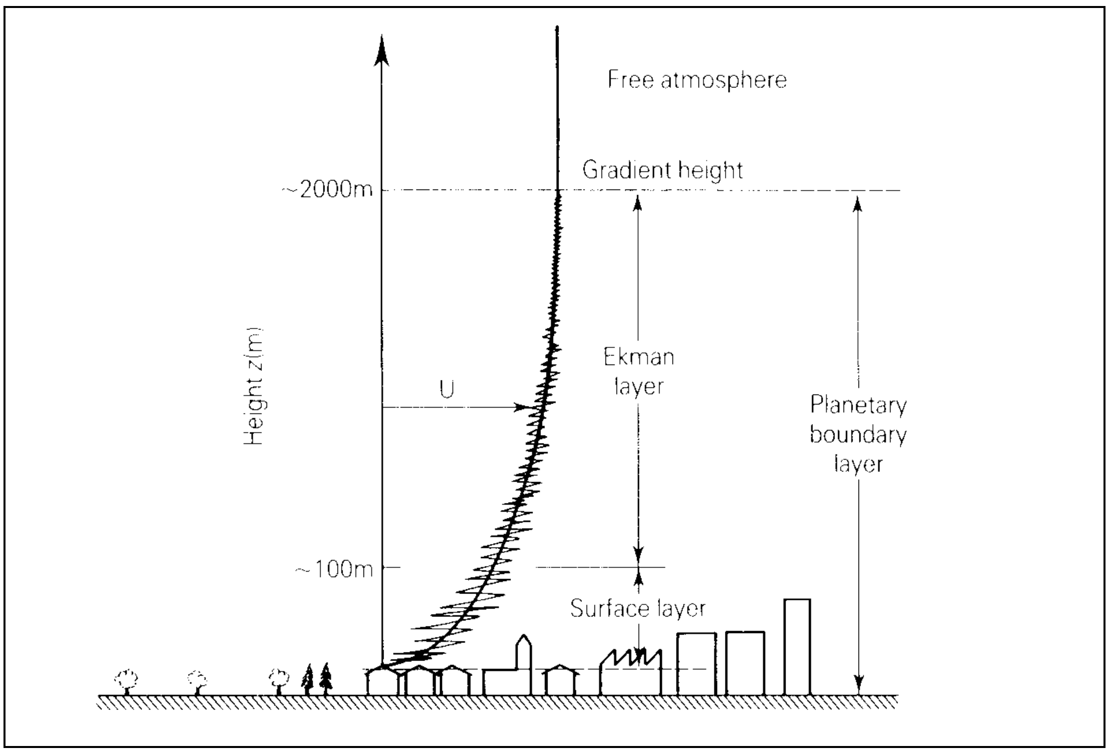

Figure 7: The atmospheric Boundary layer.

The height of this layer varies from a few hundred metres at night to as much as 2km on the most convective days. The transfer of momentum, heat and moisture, between the atmosphere and the surface, takes place within this layer.

It can be seen that the wind speed increases smoothly with height, tending to a limit which is approximately reached at the so called *gradient height*, which is at about 2000m above ground level. The variation of wind speed with height is to some extent dependent on the *atmospheric stability*. At high wind speeds needed for wind power generation, shear forces dominate. A boundary layer dominated by shear forces is called *neutrally stable*.

The Log Law

The variation of wind speed with height can be represented mathematically by a logarithmic law. The law for neutrally stable conditions is given by:

$$\bar{U}(z) = K \cdot \ln(z / z_0) \qquad [1]$$

Where $\bar{U}(z)$ is the mean wind speed at height z, K is a factor which depends on the overall wind speed and is site specific, and z_0 is known as the *surface roughness length*. The physical meaning of the factor K in equation [1] is explained in **Level 2 p13**. However, in practice, the value of this factor is rarely, if ever, evaluated for a given site.

The surface roughness length characterises the terrain and has a given value depending on the friction of the surface over which the wind passes. Typical values of z_0 are given in table 1. Equation [1] is valid up to heights of about 100m, which is sufficient for application to all current wind turbines.

Type of terrain	Z_0	a
Mud Flats, Ice	10^{-5} to $3x10^{-5}$	
Calm Sea	$2x10^{-4}$ to $3x10^{-4}$	
Sand	$2x10^{-4}$ to 10^{-3}	0.01
Mown Grass	0.001 to 0.01	
Low Grass	0.01 to 0.04	0.13
Fallow Field	0.02 to 0.03	
High Grass	0.04 to 0.1	0.19
Forest and Woodland	0.1 to 1	
Built up area, Suburb	1 to 2	0.32
City	1 to 4	

Table 1: *Typical Values of surface roughness length* z_0 *and power law exponent* ***a*** *(see later), for various types of terrain.*

It can be seen from Table 1 that the value of z_o varies considerably between terrain types. For example, the value of z_o could vary by an order of magnitude at a grassy site, between the summer and winter months, due to a growth in vegetation. This variation should be borne in mind when considering a prospective wind site.

The usual method used to determine the wind shear is to measure the mean wind speed at a given reference height z_h and to use an assumed value of z_o based on the type of terrain. This information can then be used to eliminate K and construct the new equation:

$$\frac{U(z)}{U(z_h)} = \frac{\ln(z/z_0)}{\ln(z_h/z_0)} \qquad [2]$$

Let us see how this can work in practice. Question 4 asks you to calculate the mean wind speed at several heights given that you have measured the wind speed at a reference height.

A useful approximation for evaluation of the surface roughness length is:

$$z_o = \varepsilon/30 \qquad [3]$$

where ε is the average height of roughness elements.

Level 2

The Variation of Wind Speed with Height above Ground Level:
Atmospheric Stability and the Modified Log Law

We have seen that the wind speed increases smoothly with height, tending to a limit which is approximately reached at the so called *gradient height*, which is at about 2000m above ground level. The variation of wind speed with height is to some extent dependent on the atmospheric stability. If a volume of air is displaced vertically (and adiabatically) it will tend to return to its original location if the atmosphere is stable.

If it stays at its displaced location the conditions are said to be *neutrally stable*. In an unstable atmosphere it will continue to move, due to buoyancy forces, in the direction in which it was displaced. The more unstable the conditions, the greater the mixing with the result that the velocity gradients are lower. At high wind speeds that are needed for wind power generation, shear forces dominate and the boundary layer is more or less neutrally stable.

The variation of wind speed with height can be represented mathematically by a logarithmic law which, in stability dependent form, is given by:

$$U(z) = \frac{U_*}{k}\left[1n(z/\ z_0) + \Psi_s(z/\ L_s)\right] \qquad z \rangle\rangle\ z_0 \qquad [4]$$

where U_* is the friction velocity (proportional to the square root of the turbulent shear stress, which is assumed constant in the lower boundary layer); k is known as the Von Karman constant (~0.4); z is the elevation above the ground level; and z_o is the *surface roughness length.* The stability, ψ_s , is a function of z/L_s where L_s is known as the Monin-Obukhov length. For neutral stability which is usually taken to apply to the higher wind speeds associated with wind turbine operation ψ_s is very small and can be ignored. Thus equation [1] reduces to:

$$U(z) \ = \left(\frac{U_*}{k}\right).1n\left(\frac{z}{z_0}\right) \qquad [5]$$

i.e. the constant K in main the text is equal to U_*/k. More details on this are given in **Johnson 1985.**

Since U_* is difficult to evaluate, this formula is usually rewritten in terms of a reference wind speed $U(z_h)$, at reference height, z_h, as given in the main text

$$\frac{U(z)}{U(z_h)} = \frac{\ln(z/z_0)}{\ln(z_h/z_0)} \qquad [2]$$

Equation [2] is valid up to heights of about 100m, which is sufficient for application to all current wind turbines.

The Modified Log Law

In an attempt to gain greater accuracy in the representation of wind shear, a modification has been made to the log law, based on empirical data. This so called 'modified log law' claims greater accuracy up to 300m and takes into account the Coriolis forces and the longitudinal position of the site.

The modified form of the log law that has been proposed which claims greater accuracy and applies up to about 300m:

$$U(z) = (U_* /k)[1n(z/z_0) + 5.75 z/h] \qquad [6]$$

where h = U_*/6f is the gradient height and f = 2Ωsinφ is the Coriolis parameter with φ the latitude and Ω the angular velocity of the earth.

In terms of a reference height, this becomes:

$$U(z) = U(z_r)\left[\frac{1n(z/z_o) + 5.75z/h}{1n(z_r/z_o) + 5.75 z_r/h}\right] \qquad [7]$$

The Power Law

For neutral conditions, present when wind speeds are high, a simple power law has been found to provide a reasonable fit to the data. When simple estimates of the distribution of the mean wind speed with height are required, some engineers prefer to use the empirical power law:.

$$U(z) = U(z_h)\left[\frac{z}{z_h}\right]^{\alpha} \qquad [8]$$

where α depends on the surface roughness. Typical values of α are given in table 1. To get a feel for how the log law and power law compare (i.e. equations [2] and [8]), now go through computer tutorial 1. An approximate relationship between α and z_0 over the height range of 10m to 30m is given by:

$$a = 1/\text{-}1n(Zo/15.25) \qquad [9]$$

where α depends on the surface roughness.

For z_o in the range 0.0001m and 1m, accuracy, in determining α, of the order of a few percent can be expected. Unlike the log law, there is no physical basis to equation [8].

More details are given in **Freris 1994**.

The wind resource and wind shear in open sea is discussed in Article 2. This compares how wind shear compares on land and in open sea with varying the roughness length and how wind speed over sea will effect roughness length (due to increased wave heights).

1.3.2 The Effect of Topography

Much of what we have covered in section 1.3 is only applicable to winds over flat level ground with uniform surface roughness. Equations which involve the surface roughness length z_o should be used with care where there is terrain relief or a change in roughness in the terrain. In such situations, the wind speed will vary from site to site as the wind speed profile adjusts to the influence of the terrain. The wind profile will be different from that over flat level ground with the same but uniform surface roughness.

Of the two, the effects of terrain relief will be of most relevance in wind energy. Wind turbines are often sited on top of hills to take advantage of higher wind speeds that generally prevail there which makes the problem of predicting the wind speeds at a prospective wind turbine site more difficult.

The turbulence levels over a hill are generally lower than those over level ground if the slope of the hill is not too steep. Hills with a slope of less than about 1:3 result in an increased wind speed and reduced turbulence. For slopes greater than about 1:3 we find that the wind flow starts to "separate" (i.e. the wind flow goes from a smooth "lamina" flow to a disrupted "turbulent". These ideas are discussed in Unit 4), which will result in low wind speeds with high levels of turbulence making it unsuitable to site a wind turbine.

Analysis has shown that for slopes less than about 1:3 there is a large increase in shear stress close to the ground (i.e. the wind speed increases more rapidly with height than would be the case on flat level ground). The flow field is shown schematically in figure 8.

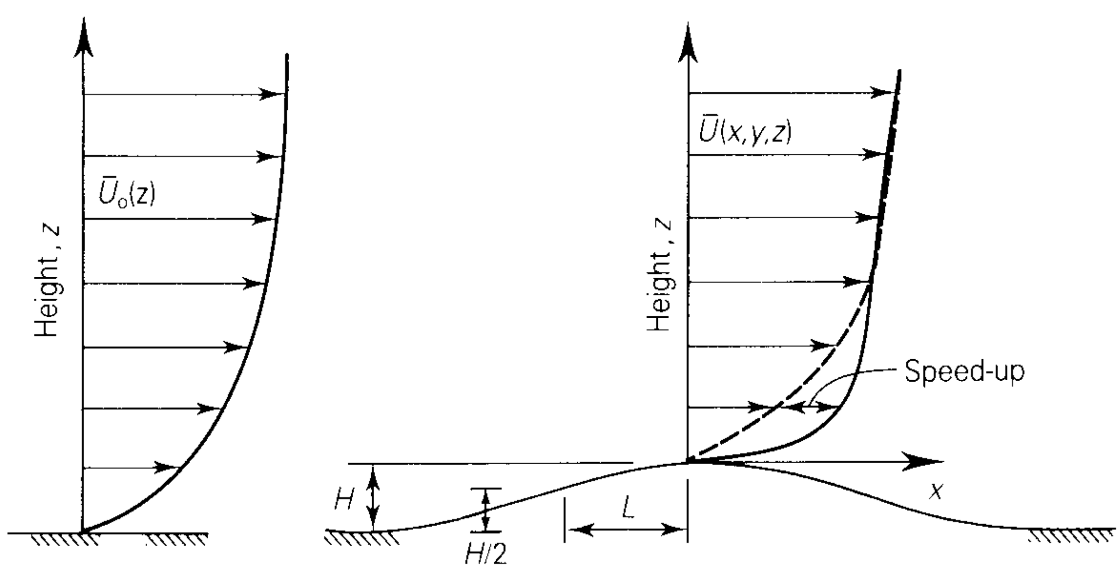

Figure 8: Diagram showing the increased wind shear on top of a small hill

1.4 Wind Speed Modelling

1.4.1 The Frequency Distribution of Wind Speed

The speed of the wind is continually changing and hence to make meaningful estimates of long term energy capture, statistical methods must be introduced. A histogram of the relative frequencies of wind speed is given in figure 9, based on small wind speed intervals (known as "bins"), at a given site measured over a period of a whole year.

The relative frequency is the proportion of wind speed measurements in each bin. It can be viewed as an estimate of the probability that a wind speed reading will be in that bin. The relative frequency is defined such that the total area under the curve has a value of unity, i.e. the probability of the wind speed being between zero and infinity is one, i.e. it is certain.

The wind speed has been measured every 10 minutes over a single year for a particular site. Each measurement has been sorted into narrow wind speed bands. This is known as *binning* the data. A bin width of 0.5m/s has been used in this instance, and for example. a measured wind speed of 7.65m/s would be placed in the 7.5 - 8m/s bin. The wind speed classes are 0-0.5m/s, 0.5-1.0m/s etc. and so the central value of each bin is taken to be 0.25m/s, 0.75m/s etc.

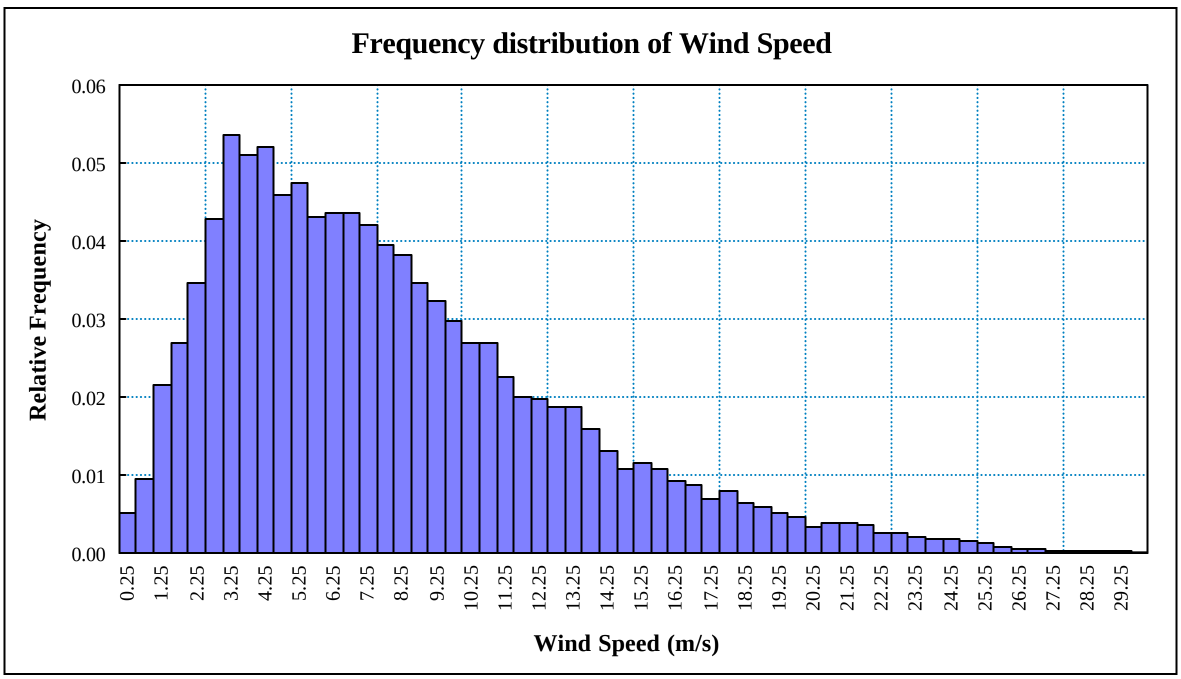

Figure 9: A wind speed frequency distribution

Two important features can be seen from the frequency distribution.

1. The mean wind speed or the average wind speed. The mean value is far higher than the 'modal' wind speed, (that is the wind speed that occurs most frequently).
2. The occasional occurrences of very high wind speeds. These have some important implications for the structural design of wind turbines (see Unit 3).

Level 2

Wind Speed Statistics

The speed of the wind is continuously changing, making it desirable to describe the wind by statistical methods. Clearly it is of little use to a wind developer to be given the wind speed at a particular instant. What is important is the *average* wind speed over a given period and how the wind speed is *distributed* around the mean value.

The mean wind speed denoted by $\overline{U}$, where the bar above the U denotes 'average', can be calculated from the formula:

$$\overline{U} = \sum_{i=1}^{n} U_i P(U_i) \qquad [10]$$

Where U_1 denotes the central wind speed value of the first bin, U_2 the second etc., and $P(U_i)$ is the probability (or relative frequency) that the wind speed is in bin *i*.

In addition to the mean wind speed, it is necessary to know how the wind speeds are distributed over the averaging period. For example, is the wind speed for a site fairly constant in time or is there a large variation? An important quantity is the *variance* of the wind speed data. This measures the amount of variation from the mean value and is given by:

$$s^2 = \sum_{i=1}^{n} U_i^2 P(U_i) - \left(\overline{U}\right)^2 \qquad [11]$$

A site where the wind is fairly constant would have a relatively low variance and a site with very changeable weather would have a relatively high variance. A commonly used measure is the square root of the variance, which is called the standard deviation and denoted simply by σ. It is attractive in having a dimension of m/s.

1.4.2 The Weibull Distribution

To calculate the energy capture from a given wind turbine at a given site and to estimate other useful parameters, such as the proportion of time the wind speed lies in a certain range, the wind speed distribution can be "modelled".

By this we mean to try and fit a mathematical function that closely resembles the wind speed distribution and which can easily be used in the required calculations.

It should be noted that modelling is generally used only in the early designed stages when long term wind speed data is unavailable (see "Measure-Correlate-Predict methods Unit 2). There is no substitution for measuring the wind speed.

Based on knowledge of wind statistics, and in particular the mean wind speed and variance at a given site, the selected probability distribution can be identified. There are a number of candidate mathematical functions which have been used. However, in wind energy analysis, the most commonly used is the Weibull distribution.

Figure 10 shows actual wind speed data together with a "best fit" Weibull distribution.

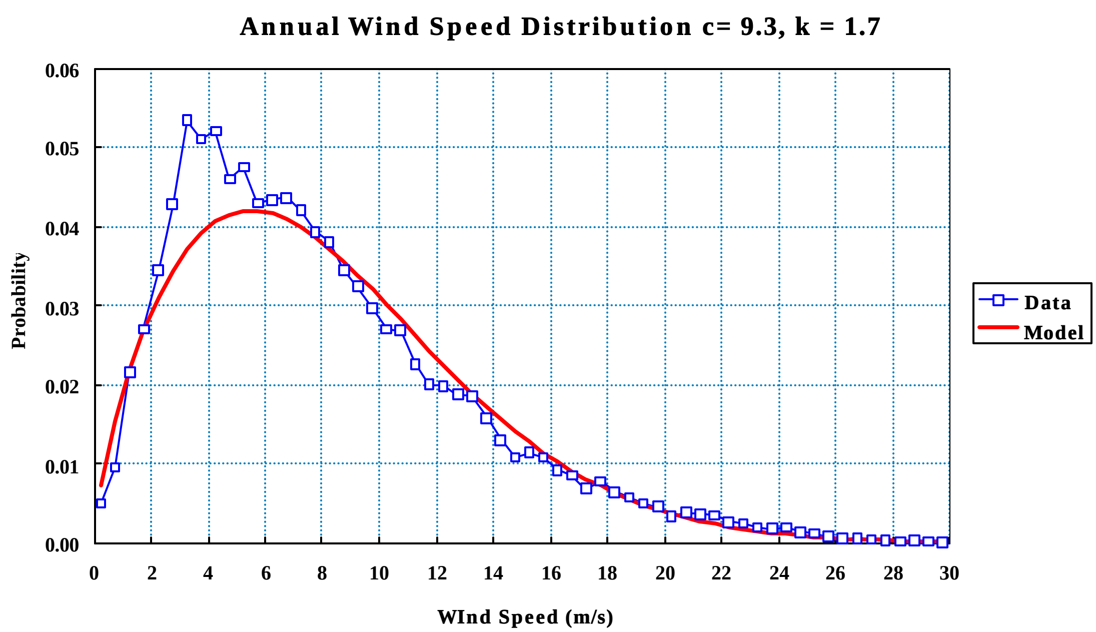

Figure 10: A Weibull distribution typical of wind regimes in the UK with a scale parameter C of 9.3m/s and a shape parameter k of 1.7.

Level 2

The Weibull Distribution

It has been found that the frequency distribution of wind speeds at most sites can be conveniently and adequately represented by the Weibull distribution function. For this function, the probability of the wind speed having a value U is given by the equation:

$$P(U) = \left(\frac{k}{C}\right)\left(\frac{U}{C}\right)^{k-1} \exp\left[-\left(\frac{U}{C}\right)^{k}\right] \qquad [12]$$

The Weibull distribution is controlled by two parameters k (the shape parameter) and C (the scaling parameter). A typical Weibull distribution is shown in figure 11.

The Weibull distribution tends to get more peaked as k gets larger with the peak moving in the direction of higher wind speeds. The parameter C scales the X-axis (wind speed) to fit different wind regimes. When the value of k is equal to 2 the Weibull distribution reduces to the one parameter *Rayleigh distribution*. This is easier to use, but only applicable to certain wind regimes.

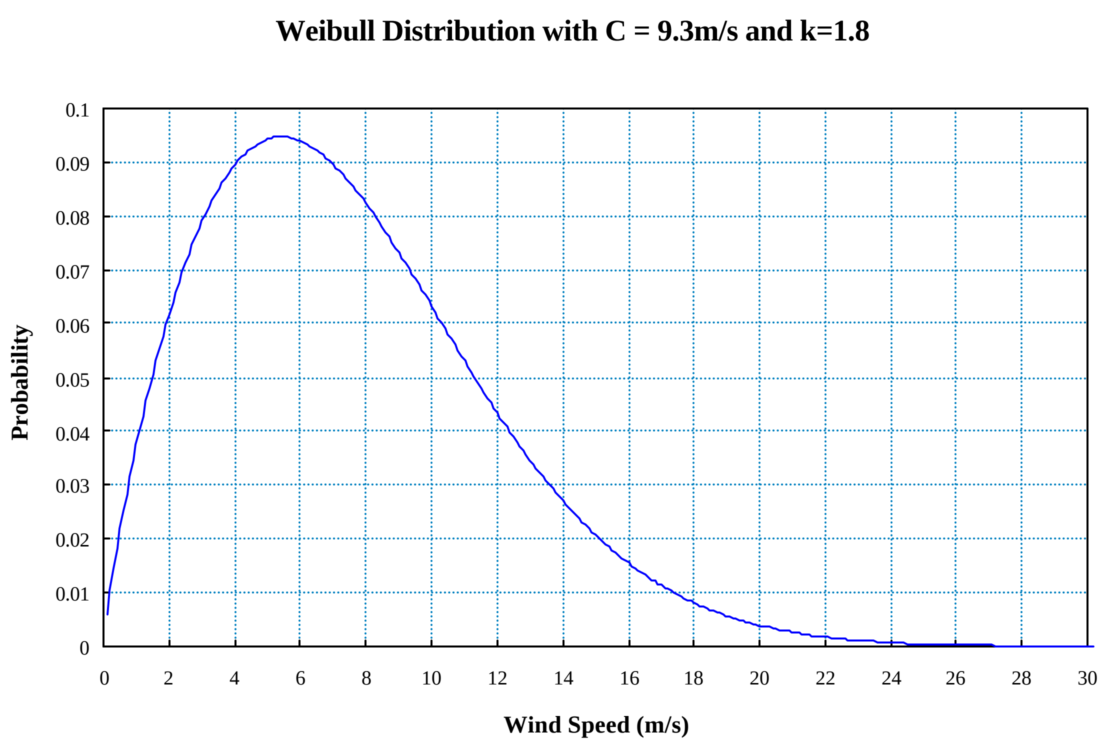

Figure 11: A Weibull distribution typical of wind regimes in the UK with a scale parameter C of 9.3m/s and a shape parameter k of 1.8.

Weibull Distribution at Low and High Wind Speeds.

For k greater than 1 (which is always the case for reasonable wind sites) it can be seen from equation [12] that the Weibull distribution gives a zero probability of having a wind speed of zero. Clearly this is not accurate since there will always be periods of calm i.e. when U = 0.

However, this is not a problem because, for reasons discussed in Unit 5, the turbine will not be running at very low wind speeds and so this inaccuracy will not affect the estimate of annual energy capture.

The Weibull Distribution tends to zero at high wind speeds. This means that there will be a non-zero (albeit very small) probability of obtaining all wind speeds up to infinity. This inaccuracy at high wind speeds is not a problem because at high wind speeds the wind turbine is shut down.

The Cumulative Weibull Distribution

It is often required to know the probability of the wind speed at a particular site lying in a particular range (to optimise the turbine for a particular site for example). This can be calculated by using the cumulative Weibull distribution.

The cumulative Weibull distribution is obtained by integrating the Weibull distribution between zero and the particular wind speed of interest. This gives the probability that the wind speed is greater than U as the following:

$$F(U) = \exp\left\{-\left(\frac{U}{C}\right)^k\right\} \qquad [15]$$

This formula can be used to calculate the probability of the wind speed falling in a given range. For any two values of wind speed, U_1 and U_2, the probability of the wind speed being between U_1 and U_2 is simply $F(U_1) - F(U_2)$. Clearly $F(0) = 1$ and $F(\infty) = 0$. For most sites $F(30) \approx 0$, hence wind speeds greater than about 30m/s are of little importance for annual energy capture calculations.

A plot of the cumulative Weibull distribution for the values of C and k for the site in computer assignment 3 is shown in figure 12.

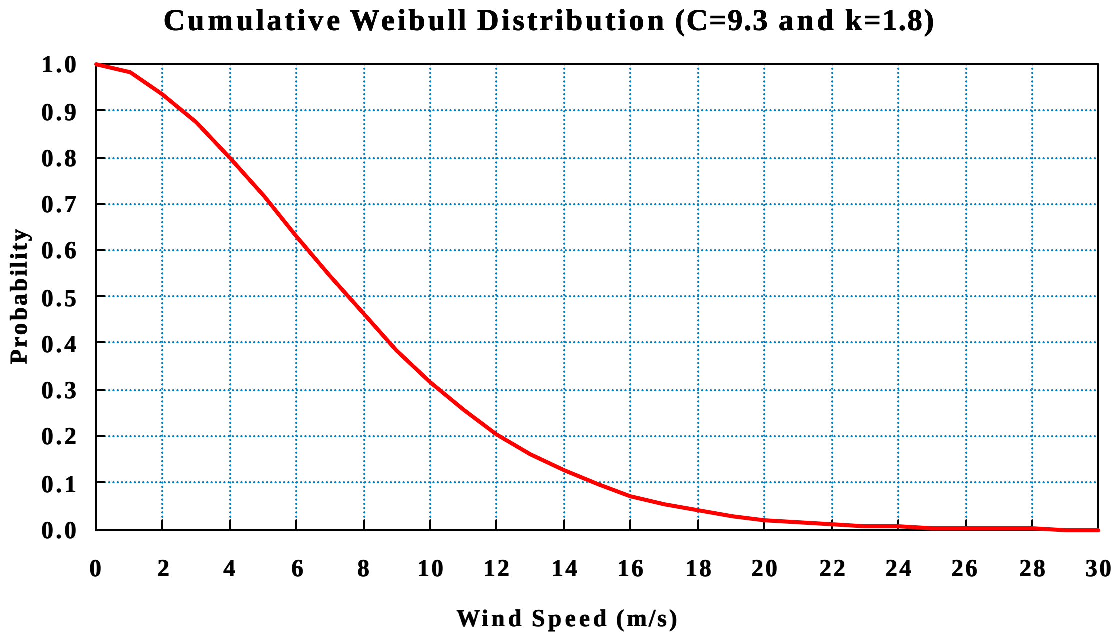

Figure 12: Shows the probability that the wind speed is greater than a particular wind speed U. It is clear that the curve must tend to zero as wind speed increases.

The cumulative Weibull distribution also gives a method for estimating the Weibull parameters by plotting the double logarithm of the cumulative probability against the log of the wind speed.

Taking logarithms twice on both sides of equation [15] gives:

$$\ln(-\ln(F(U)) = k \ln(U) - k \ln(C). \qquad [16]$$

Hence if 1n(-1nF(U)) is plotted against 1n(U), where F(U) is calculated from the data for a wide range of values of U, then the result should be approximately a straight line with k as the gradient and an intercept on the Y-axis at -k.1n(C).

Care must taken using this method however because small errors in data are amplified by the use of two logarithmic functions in equation 16.

The result of such an analysis showing that the Weibull distribution is a reasonable fit to wind speed data is shown in figure 13.

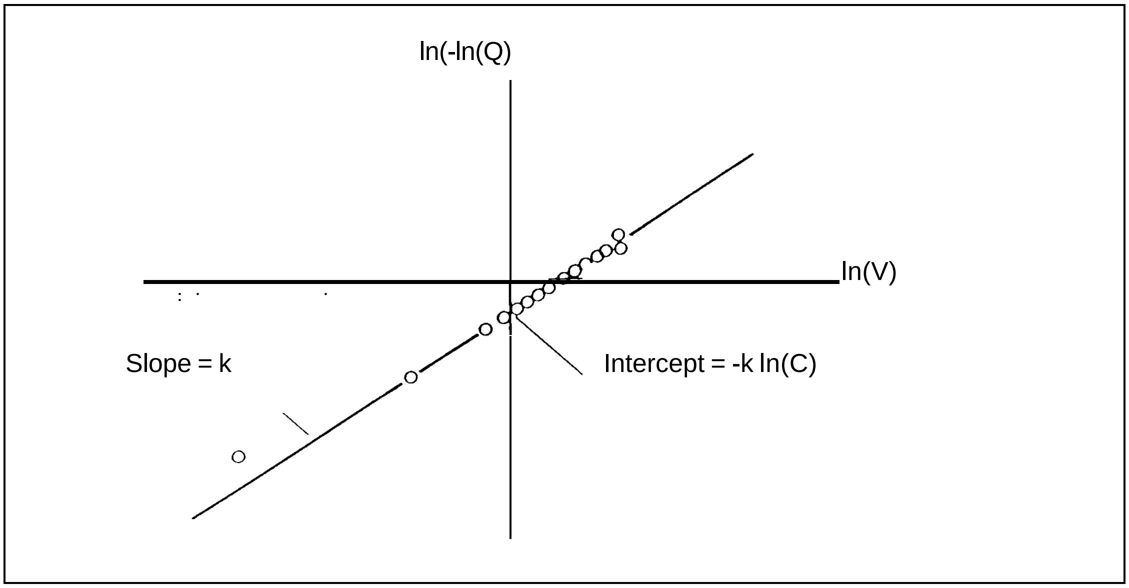

Figure 13: Fitting to identify Weibull parameters

Although widely used, linear regression applied to logarithms and double logarithms of the dependent and independent variables will not provide the best choice of parameters k and C.

A more rigorous approach to parameter estimation uses maximum likelihood techniques. These are recommended if it is important to have great accuracy.

In many situations however rough estimates of k and C will suffice. In these circumstances the following approximate relations are very useful.

Estimation of Weibull parameters

For a given site with measured mean wind speed and variance, values of k and C can be estimated by the following relations:

$$k \cong 3C^3 p^{1/2}/(2\overline{U}^3) \quad for\,(1.8 \le k2.3) \qquad [17]$$

and

$$C \approx \left(\frac{2}{\sqrt{\pi}}\right)\bar{U} \approx 1.13\,\bar{U} \qquad [18]$$

for 1.6 < k <3.0

Provided the ranges indicated are adhered to the accuracy of these formula is within 1%.

Another approximate relation, which does not depend on knowing the mean of U^3, is:

$$k \approx \left(\frac{\sigma}{\bar{U}}\right)^{-1.086} \qquad [19]$$

To get a feel for how the Weibull distribution can be used to model a real wind speed distribution and to understand the physical meanings of the two parameters k and C go through computer assignment 3.

In special cases the wind speed at one point will be approximately proportional to the wind speed at another point. For example if the points are two heights at a given location for which the simple log law (neutral stability) applies.

If $U_1 = aU_2$, then $k_1 = k_2$ and $C_2 = aC_1$

Calculation of Energy Yield from a Wind Turbine

It is straightforward to calculate the energy yield from a wind turbine to be placed on a given site, using the Weibull parameters and the wind turbine power curve.

The average power from the wind turbine, assuming 100% reliability, is given by

$$\overline{P} = \int_{u=0}^{\infty} p(U)\, P(U)\, dU \qquad [20]$$

where P(U) is the power output from the wind turbine at wind speed, U and p(u) is the probability given by equation (12). This integral will in general have to be evaluated numerically.

If the record of hourly mean wind speeds from the site is available this can be used directly with the power curve to calculate the energy yield which would have occurred had the turbine experienced that specific wind history.

Estimating the Optimum Turbine Rating using the Weibull Distribution

Wind speeds close to the average site wind speed occur more frequently than higher wind speeds, however they contain less energy. Clearly there is a trade-off between having a low design wind speed and running close to that design speed most of the time, and having a higher design wind speed which would give a higher power output, but this will be achieved a smaller fraction of the time.

We shall see in later units that there is always a "cut-in" wind speed for a given turbine, below which the turbine will not generate any power. Each turbine will produce its designed power at a particular 'rated wind speed'. At higher wind speeds than this, various mechanisms have been devised to keep the turbine output at the rated power (see unit 5) by extracting a smaller fraction of the energy contained in the wind.

The wind speed U_{me} that will give the most energy is given by the maximum value of the product $U^3.P(U)$ i.e. the cube of the wind speed multiplied by the relative frequency of that wind speed. By modelling the wind speed data by the Weibull distribution U_{me} can be estimated and is given by:

$$U_{me} = C\left(\frac{k+2}{k}\right)^{1/k} \qquad [21]$$

U_{me} will typically be between 1.4 and 1.7 times greater than $\bar{U}$ depending on the wind regime. A derivation of equation [24] is given in the book "Wind Energy Systems", by G.L. Johnson, 1985.

1.5 Turbulence

Short term variations in wind speed are also important in the design and evaluation of wind turbines. For simplicity we can regard the instantaneous wind speed U(t) as a quasi-steady component $\bar{U}$ and a turbulent fluctuations u(t)about this mean wind value i.e.

$$U(t) = \bar{U} + u(t) \qquad [25]$$

The quasi-steady state value is given by averaging over an appropriate period of time.

Over this same period we characterise the variability of the wind by the variance of U(t), which is equal to the mean square value of u(t) in equation [25].

An indication of the gustiness at a given site is provided by a parameter, known as the turbulence intensity, denoted I. This is defined as

$$I = \sigma / \bar{U} \qquad [26]$$

The averaging period used is often unstated, but this is inadvisable since the calculated value will depend to some extent on the length of this time period. The reason for this is that the wind speed is not exactly stationary in the statistical sense; in other words there is a tendency for statistical parameters such as the mean and standard deviation to vary in time. It is good practice therefore, always to state the averaging period that has been used in the calculation of turbulence intensity, according to equation [26]. Ten minutes is an attractive averaging time since it lies within the spectral gap.

It can be shown that the mean value of the wind speed cubed is equal to the mean wind speed cubed by the relationship

$$\overline{(U^3)} = (\bar{U})^3 (1 + 3I^2) \qquad [27]$$

Although this implies (theoretically) that a more turbulent wind gives a greater energy, the wind turbine may not be able to react quickly enough to make use of it

1.6 What's in the Next Unit?

We have seen in this unit how winds arise on a global scale and how the wind is affected by local factors. We have also seen how the wind speed varies with height above ground level and how the wind speed varies in time. The unit then concentrated on how the long term wind speed distribution can be modelled mathematically by the Weibull distribution.

In the next unit we shall learn how wind speed and direction are measured and how the data can be presented in a useful way.

Unit 2 then focuses on how the long term wind speed can be determined using meteorological data and the method of "Measure-Correlate-Predict".

Finally, physical methods and computational models which can be used to estimate the wind resource at a particular site are described.

Notation and Units

Symbol	Quantity	Units
$\bar{U}$	Annual Mean Wind Speed	m/s
U(t)	Wind Speed at a given Instant	m/s
Z	Height above ground level	m
Z_h	Reference height above ground level	m
Z_0	Surface Roughness Length	m
α	Power Law Exponent	None
σ^2	Variance of the Wind Speed Distribution	m^2/s^2
k	The Shape Parameter of the Weibull Distribution	None
C	The Scaling Parameter of the Weibull Distribution	m/s
ρ	The density of air = 1.225 Kgm^{-3}	kg/m^3
I	Turbulence Intensity	None